Be Prepared
for the

Calculus Exam

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Appendix: Calculator Skills

You should practice several calculator skills prior to the AP Exam. A few examples follow, with calculator-assisted solutions for the TI-83, TI-84, TI-86, and TI-89 models. There are other acceptable calculator methods to solve these problems. If your calculator model does not match one of the models presented, consult your user's manual to solve the examples.

A.1. Graphing a Function

This is the simplest calculator skill required on the exam. Usually, the hardest part is making sure you enter the function correctly on your calculator, and that you choose a suitable viewing window. Be sure to check that the parentheses that enclose function arguments (as in `sin(X)`) are properly matched.

> **Be sure that your calculator is set to the `Radian` mode when you take the exam. (To set the mode, go to the `MODE` menu.)**

You will see several graphing examples in the following sections.

A.2. Solving an Equation

Graphing calculators offer several methods for solving an equation.

> **The easiest approach is to enter the functions for each side of the equation, graph the two functions, and use the `intersect` command from the `GRAPH` screen. However, this method may <u>not</u> give the required accuracy.**

If you are looking for a root of an equation or a zero of a function, graph the function and use the `Root` or `Zero` command to find the *x*-intercept of the graph. The TI-83 and TI-84 come with an `EQUATION SOLVER`, and the TI-86 has a `SOLVER` environment where you can find zeroes using arbitrary names of variables (while in the graphing environment, the independent variable is usually named `X` or `T`).

Before using the EQUATION SOLVER environment, you should graph the equation in order to get an idea how many roots there are, and where those roots are located. The one you find with the EQUATION SOLVER might not be the one you need. Using a "seed" value for the variable that is close to the root you are looking for usually works.

Example 1

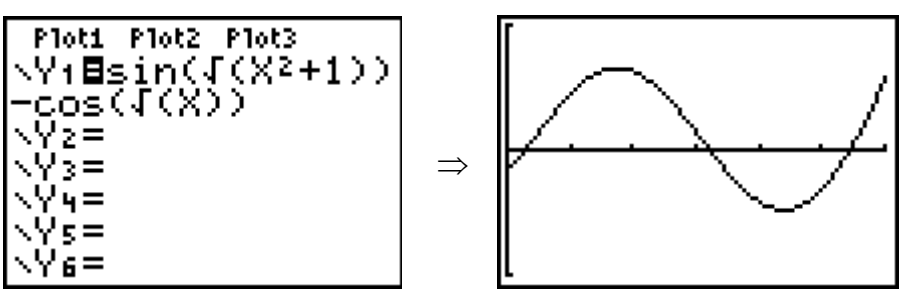
The derivative of a function f is given by $f'(x) = \sin\left(\sqrt{x^2+1}\right) - \cos\left(\sqrt{x}\right)$. Find all the values of x in the open interval (0, 6) where f has a local minimum.

TI-83 or TI-84 Solution

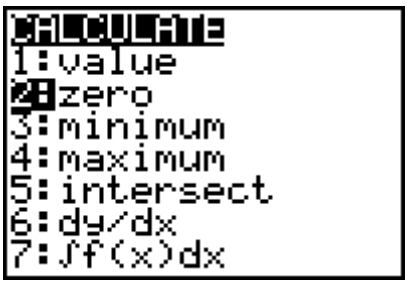
We need to determine where the derivative changes sign from negative to positive. It is a good idea to graph the given derivative first, especially when you are given a domain of values for the independent variable. Then use the Zero command from the CALC menu.

Here are the necessary steps. First press WINDOW to set your viewing window:

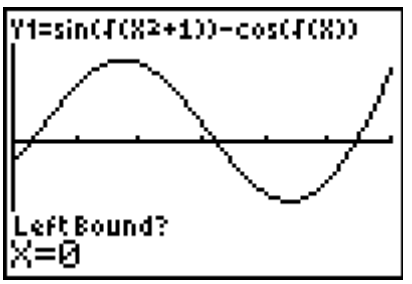
Note that the choices of Xmin and Xmax correspond to the interval given in the problem. Press Y= and set Y_1 to $f'(x)$, taking care that your parentheses are properly matched. Then press GRAPH to plot the graph:



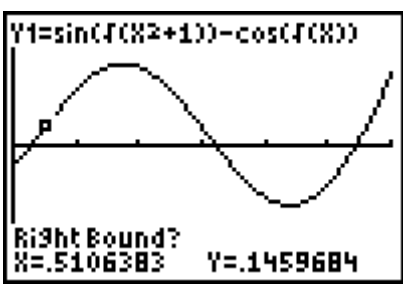
You can see two points where the derivative changes sign from negative to positive. Go to the CALC menu (2nd+TRACE), and choose 2:zero:



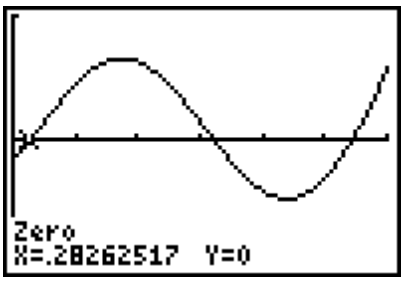
You will be prompted to enter a left bound for the zero. You can either use the arrow keys to move the cursor to the left of the leftmost zero, or simply enter a number for *x* using the keypad. You have to be sure the number is to the left of the zero you are looking for (but still in the viewing window). Type in 0 for *x*:



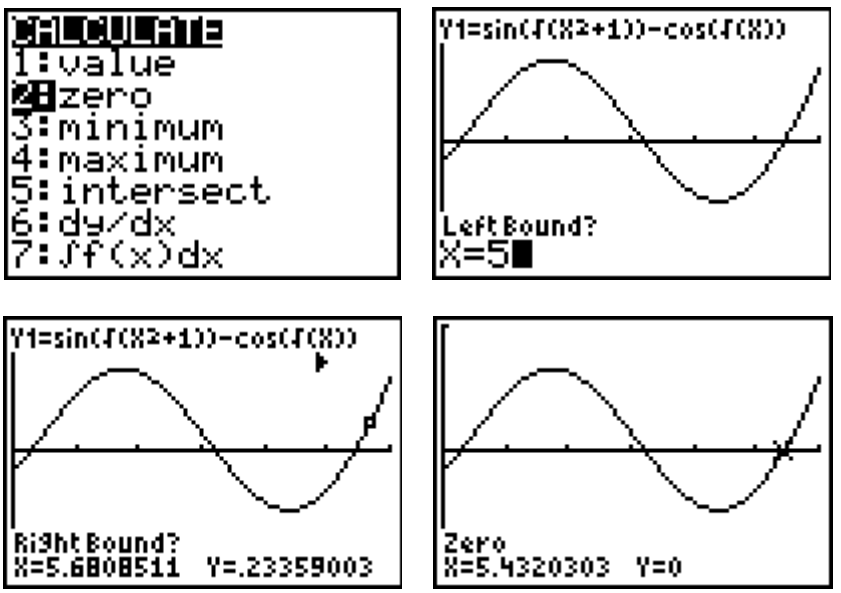
Then either move the cursor over to the right of the zero, or enter a value of *x* that you know is to the right of the zero (but not past the next zero). Suppose you use the cursor (and press ENTER when you are satisfied with its position):



Your calculator then prompts you: `Guess?` If you've chosen the left and right bounds so that only one root is between them, then just press `ENTER`. The zero will then be calculated, and stored into the variable `X`:



Repeat the process to find the other zero where the derivative changes sign from negative to positive:

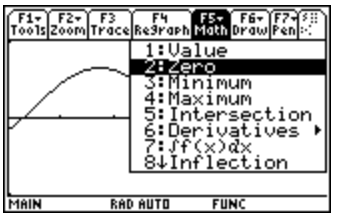


The answers (accurate to three digits to the right of the decimal) are $x \approx .283$ and $x \approx 5.432$.

> **For the exam, you must use the `MATH/zero` command from the `CALC` menu on the `Graph` screen, (or the `EQUATION SOLVER` environment from the HOME screen discussed below) to find a root or an intersection point. Just tracing along a graph to find roots or intersection points may not give you the required accuracy.**

TI-86 or TI-89 Solution

The same basic procedure works for the TI-86 or TI-89. However, to find a zero of a function on the TI-86, use the `Root` command on the `MATH` menu on the `GRAPH` screen. On the TI-86, the `MATH` menu is on the second page of commands on the `GRAPH` screen. On the TI-89, press the `F5` key and use the `Zero` command:



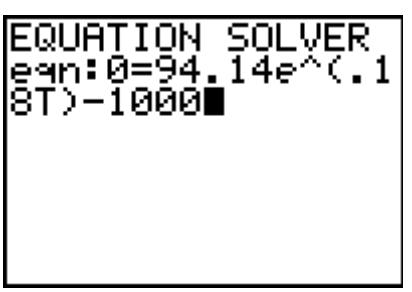
The following example illustrates the use of the `EQUATION SOLVER` environment on the TI-83 or TI-84.

Example 2

The number of bees in a colony is given by $B(t) = 523e^{0.18t}$, where t is the number of days since the colony was established. The derivative of $B(t)$ is given by $B'(t) = 94.14e^{0.18t}$. On what day is the number of bees in the colony increasing at the rate of 1000 bees per day?

TI-83 or TI-84 Solution

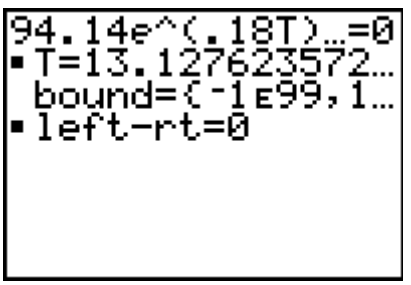
Press `MATH`, then `0` (or ALPHA B on the TI-84 with MathPrint*) to bring up the `EQUATION SOLVER`. (We are using the Solver environment for this problem because we have no way of knowing what window settings we might need to use the graph environment.) Press the arrow-up key to get into the equation editing mode (identified by the `eqn:0=` line), if you are not already there.



* MathPrint is a feature in TI-84 calculators with the operating system version OS2.53MP or higher, introduced in 2010.

Note that the equation we've entered is equivalent to $0 = B'(t) - 1000$. Solving the equation answers the question posed in the example. Press ENTER to return to the main solver screen. Use the arrow keys to position the cursor on the line that starts with T=, then press ALPHA, then ENTER to solve for *T*.

Be patient: this operation may take a few seconds.



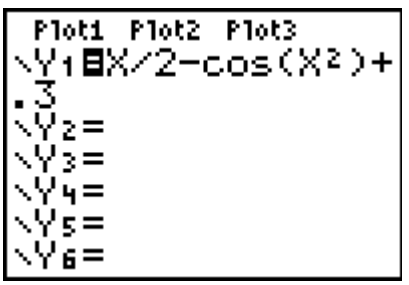
$T \approx 13.128$; the answer is on the 14th day.

Example 3

The derivative of a function *g* is given by $g'(x) = \frac{x}{2} - \cos\left(x^2\right) + 0.3$. What is the *x*-coordinate of a local maximum point on the graph of *g*?

TI-83 or TI-84 Solution

Enter $g'(x)$ into Y_1, and graph it in the ZDecimal window.



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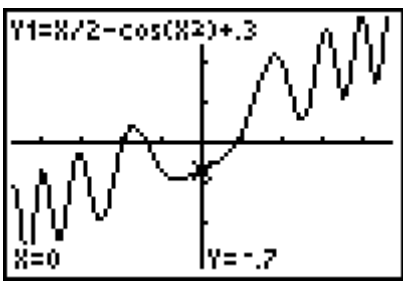
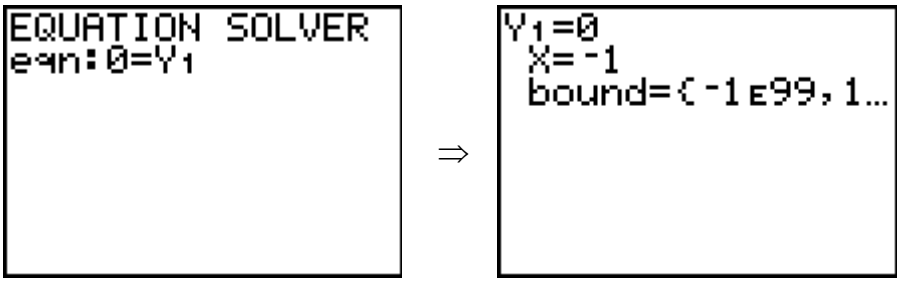
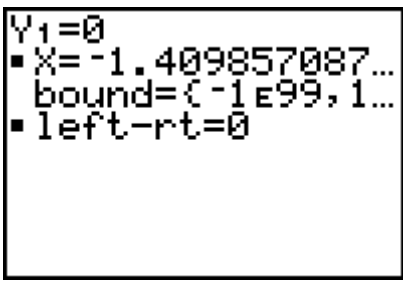


We need to find a zero where $g'(x)$ changes sign from positive to negative. From the graph, this zero appears to be near $x = -1$. So, on the Solver menu, enter the equation as shown, then enter a "seed" value of -1 for X:



Press ALPHA, then ENTER to solve for X:



The local maximum occurs at the point where $x = -1.410$.

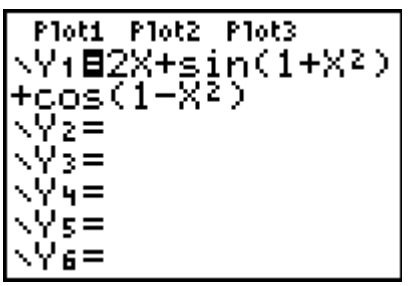
A.3. Evaluating a Derivative at a Point

Example 4

Find the slope of the line tangent to the graph of $y = 2x + \sin(1+x^2) + \cos(1-x^2)$ at the point where the graph crosses the *x*-axis.

Solution

First enter the function into Y_1:



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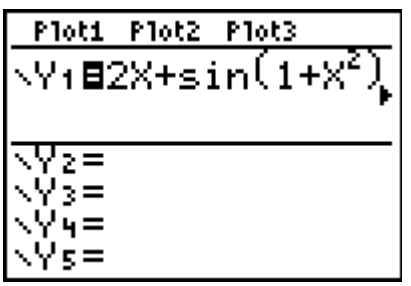
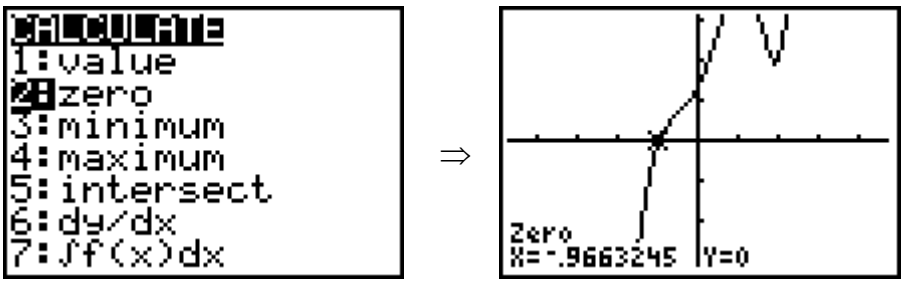
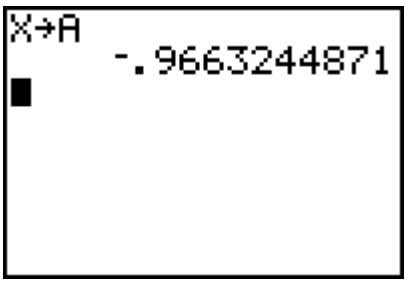


TI-83/84 **84Plus with MathPrint**

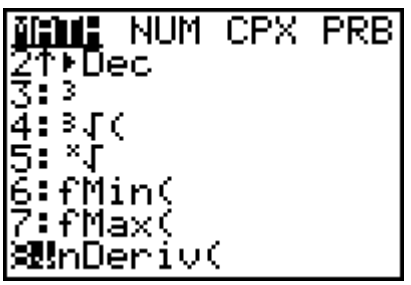
Then use the procedure from the previous section to find the zero of a function:

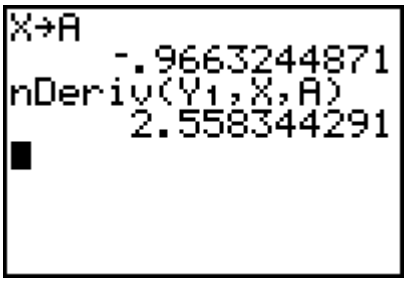


The zero is at $x = -.9663245$. Immediately after finding that zero, return to the HOME screen and use the STO➔ command to store the calculator's current value of the x-coordinate from the graph, which is X on the TI-83 or TI-84 (or x on the TI-86, or xc on the TI-89) into a variable, like A on the TI-83, TI-84, or TI-86 (or a on the TI-89):



Then calculate the numerical derivative of Y_1 at A. The numerical derivative command is number 8 on the MATH menu:





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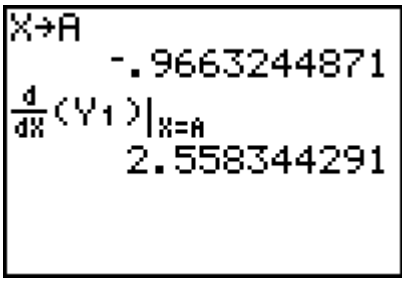
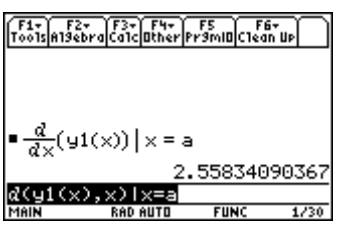


TI-83/84 **84Plus with MathPrint**

On the TI-83 or TI-84, the variable Y_1 must be pulled off of the Y-VARS submenu on the VARS menu, Choice 1. On the TI-86 or TI-89, you can just type y1 from the keypad. On the TI-89, you can use the derivative command located at 2nd 8 on the keypad, in conjunction with the | command to evaluate the derivative:



A.4. Evaluating an Integral Numerically

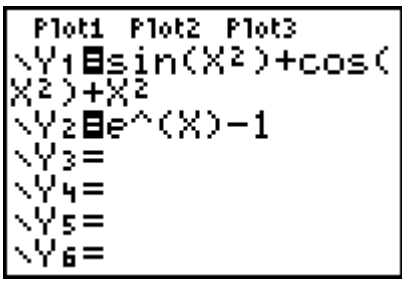
On the open-calculator free-response part of the AP exam (Section II, Part A), always use your calculator when you need to evaluate a definite integral. You don't get "extra credit" for evaluating an integral by first finding an antiderivative.

Example 5

Find the area of the region in the first quadrant bounded by the graphs of $f(x) = \sin(x^2) + \cos(x^2) + x^2$, $g(x) = e^x - 1$, and the y-axis.

Solution

First, take a look at the graphs:



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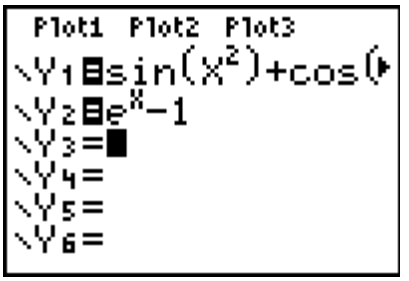
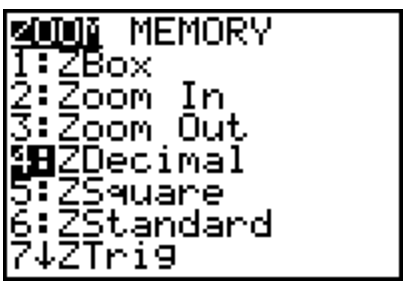


TI-83/84 **84Plus with MathPrint**



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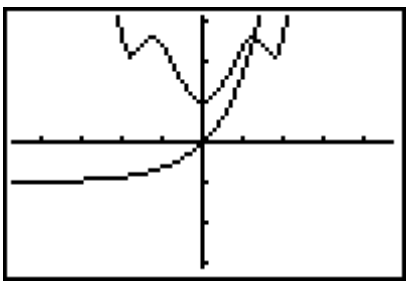
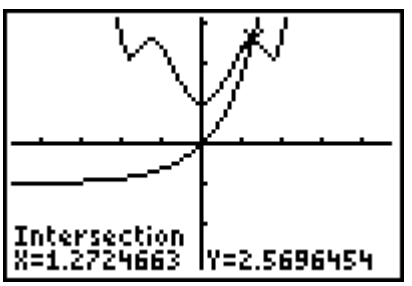


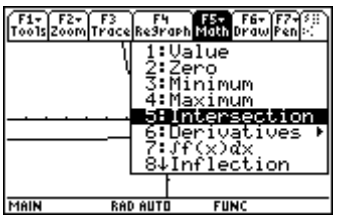
To find the area, we must first find where the curves intersect.

On the TI-83 or TI-84, press `CALC` (`2nd+TRACE`), then choose 5 to use the `intersect` command. Press `ENTER` three times to select the first curve, second curve, and "guess" for the intersection point:



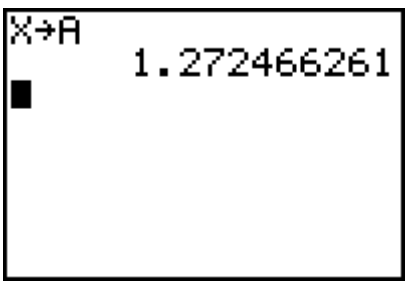
(You can use the up and down arrow keys to select from among several graphed functions the two you want, and the left and right arrow keys to change the "guess" to a value that will result in the intersection you want to find.)

On the TI-86, ISECT is on the second page of the MATH menu. The screen below shows where to find `Intersection` on the TI-89.



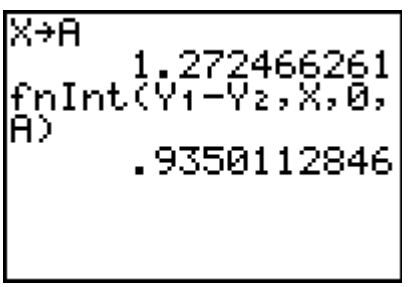
On the TI-89, you need to choose lower and upper bounds for the *x*-coordinate of the point of intersection.

After you've found the intersection, go to the HOME screen. Then immediately store the current value of `X` into the variable `A` (on the TI-86 use `x`, and on the TI-89 use `xc`) by pressing `X STO➔ A`:



Now you're ready to evaluate an integral to get the area. Area = $\int_0^A Y_1(X) - Y_2(X)\,dx$.

Enter the integral as follows:



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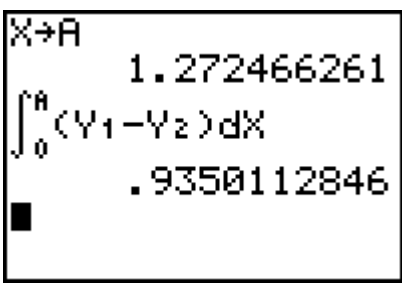
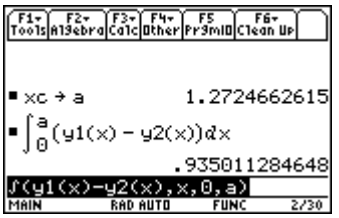


TI-83/84 **84Plus with MathPrint**

On the TI-83 or TI-84, `fnInt` is option 9 in the `MATH` menu, and you must use the `Y-VARS` submenu on the `VARS` menu, Choices 1 and 2, to enter Y_1 and Y_2. On the TI-86, `fnInt` is on the `CALC` menu.

On the TI-89, it is a good idea to clear out all your single letter variables before starting a problem like this. You do that by pressing `2nd F1 ENTER Clear a-z`:

You can use the integral key found on the keypad:



On the TI-89, you should press ◆ `ENTER` after typing the integral to force the calculator to give you a numeric (approximate) result.