2005

1. $x<3 \Rightarrow |x-3| = 3-x \Rightarrow \lim_{x\to 3^-} \frac{|x-3|}{3-x} = \lim_{x\to 3^-} \frac{3-x}{3-x} = \boxed{1}$.

---

2. At $x=4$, the slope of the tangent line is $2x-4\big|_{x=4} = 4$, $y = 4^2 - 4\cdot 4 - 5 = -5$, an equation of the tangent line is $y+5 = 4(x-4)$. The line of simmetry is $x=2$. At the intersection, $y+5 = 4(2-4) \Rightarrow y = -13$. The coordinates of the intersection points are $\boxed{x=2,\ y=-13}$.

---

3. $y = f(2) + f'(2)(x-2) \Rightarrow y = 7-3(x-2) \Rightarrow \boxed{y = -3x+13}$.

---

4. $y' = 3x^2 - 12x + 9 = 3(x-1)(x-3);\ y'' = 6x-12 = 6(x-2)$. On the interval $1<x<2,\ y'<0$ and $y''<0$. Therefore, the curve is $\boxed{\text{decreasing and concave down}}$.

---

5. Since $\sqrt[3]{x}$ is increasing, $f(x)$ reaches the minimum at the same x as $x^2 + 4ax + 12a^2$, that is, at $x=-2a$. $f(-2a) = \sqrt[3]{4a^2 - 8a^2 + 12a^2} = \sqrt[3]{8a^2} = \boxed{2\sqrt[3]{a^2}}$.

---

6. A function is defined for all real numbers and has the following property: $f(x+h) - f(x) = 4x^2h + 2xh - 6x^3h^2$. Find

$$f'(-3) = \lim_{h\to 0} \frac{f(-3+h) - f(-3)}{h} = \lim_{h\to 0} \frac{4(-3)^2 h + 2(-3)h + 6(-3)^3 h^2}{h} =$$

$4\cdot 9 - 6 = \boxed{30}$.

7.■ $3x^2 - 12x = -9 \Rightarrow x=3$ or $x=1$. For $x = 3$, $3^3 - 6\cdot 3^2 = -9\cdot 3 + k \Rightarrow k=0$. For $x = 1$, $1^3 - 6\cdot 1^2 = -9 + k \Rightarrow \boxed{k=4}$.

8.■ $x\cdot 7.5+2y\cdot 5=9000 \Rightarrow y=900-.75x \Rightarrow xy=x(900-.75x)$ reaches maximum at $x=\dfrac{900}{1.5}=600 \Rightarrow y=450$. $\boxed{600 \text{ by } 450}$.

---

9. $\left.\begin{array}{l}\lim\limits_{x\to-\infty}\left(10-2^{x}\right)=10\\ \lim\limits_{x\to-\infty}\left(10+2^{-x}\right)=\infty\end{array}\right\} \Rightarrow \lim\limits_{x\to-\infty}\dfrac{10-2^{x}}{10+2^{-x}}=\boxed{0}$.

---

10. $y(p)=ap^2;\ y'(p)=2ap \Rightarrow$ an equation of the tangent line is $y=ap^2+2ap(x-p)$. Its x-intercept is when $ap^2+2ap(x-p)=0 \Rightarrow \boxed{x=\dfrac{p}{2}}$.

---

11. $h'(5)=f(5)g'(5)+g(5)f'(5)=3\cdot 5+(-2)\cdot 4=\boxed{7}$.

---

12. $h'(4)=f'\left(g(4)\right)g'(4)=f'(3)g'(4)=(-5)\cdot 9=\boxed{-45}$.

---

13. $f(2.01)\approx f(2)+f'(2)\cdot 0.01=7-3\cdot 0.01=\boxed{6.97}$.

---

14. $y'=6x^2-6(k+1)x+6k=6(x-1)(x-k)$. $y''=12x-6(k+1)=12\left(x-\dfrac{k+1}{2}\right)$.

For $1<x<k,\ x-1>0;\ x-k<0 \Rightarrow y'<0$. If $1<x<\dfrac{k+1}{2}$, $y''<0$. If $\dfrac{k+1}{2}<x<k$, $y''>0$. $\boxed{y' \text{ is negative, and } y'' \text{ is first negative, then positive}}$.

15.■ $V=\left(\sqrt{\dfrac{A}{6}}\right)^{3}=\left(\dfrac{A}{6}\right)^{\frac{3}{2}} \Rightarrow \dfrac{dV}{dt}=\left(\dfrac{1}{6}\right)^{\frac{3}{2}}\dfrac{3}{2}A^{\frac{1}{2}}\dfrac{dA}{dt}=\left(\dfrac{1}{6}\right)^{\frac{3}{2}}\dfrac{3}{2}108^{\frac{1}{2}}9\sqrt{2}=$

$\dfrac{27}{2}\left(\dfrac{108\cdot 2}{216}\right)^{\frac{1}{2}}=\boxed{\dfrac{27}{2}}$.

16.■ $x_A = 9t - \frac{0.2t^2}{2}$. $x_B = 9t + \frac{0.1t^2}{2} - 7.35$. $x_A = x_B \Rightarrow \frac{0.3t^2}{2} = 7.35 \Rightarrow$

$t^2 = 49 \Rightarrow t = 7 \Rightarrow x_A = 9 \cdot 7 - 0.2\frac{49}{2} = 63 - 4.9 = \boxed{58.1}$.

---

17. $f'(3) \approx \frac{f(3.03) - f(3)}{0.03} = \frac{8.168 - 8}{0.03} = \frac{16.8}{3} = \boxed{5.6}$.

---

18. The graph of $f(x)$ is concave down when $f'(x)$ is decreasing, that is when $\boxed{x < -4 \text{ or } 0 < x < 4}$.

---

19. $f'(2) = 2g(2)g'(2) = 2 \cdot 8 \cdot 3 = \boxed{48}$.

---

20. $\int_4^5 f(x)\,dx = \int_0^5 f(x)\,dx - \int_0^4 f(x)\,dx = 9 - 10 = -1$.

$\int_5^7 f(x)\,dx = \int_4^7 f(x)\,dx - \int_4^5 f(x)\,dx = 1 - (-1) = \boxed{2}$.

---

21. $u = x^2 + 1 \Rightarrow u(1) = 2;\ u(2) = 5;\ du = 2x\,dx \Rightarrow$

$\int_1^2 \frac{x^2}{x^2+1}\,dx = \int_2^5 \frac{x^2}{u}\frac{du}{2x} = \int_2^5 \frac{x}{2u}\,du = \boxed{\int_2^5 \frac{\sqrt{u-1}}{2u}\,du}$.

---

22. Average area $= \frac{\int_3^6 \pi r^2\,dr}{6-3} = \frac{1}{3} \cdot \left.\frac{\pi r^3}{3}\right|_3^6 = \frac{\pi}{3}(72 - 9) = \boxed{21\pi}$.

23.■ $N = \int_2^3 \left(3t^2 + 6t\right)dt = \left.\left(t^3 + 3t^2\right)\right|_2^3 = (27 + 27) - (8 + 12) = \boxed{34}$.

24.■ There are four regions of equal area. The total is

$$4\int_0^{\frac{\pi}{2}}\left(\sin x-\tan\frac{x}{2}\right)dx=4\left[-\cos x+2\ln\left|\cos\frac{x}{2}\right|\right]_0^{\frac{\pi}{2}}=4\left[\ln\left(\frac{1}{\sqrt{2}}\right)^2+1\right]=\boxed{4(1-\ln 2)}.$$

---

25. Since e^x is a continuous function, $\lim\limits_{x\to 2} f(x)=\lim\limits_{x\to 2} e^{\ln f(x)}=e^{\lim\limits_{x\to 2}\ln f(x)}=\boxed{e}$.

---

26. $C(0,\, c)$ is equidistant from $A\left(a,\, a^2\right)$ and $V\left(0,\, 0\right)$, so $(a-0)^2+\left(a^2-c\right)^2=c^2 \Rightarrow$

$a^2+a^4-2a^2c+c^2=c^2 \Rightarrow 1+a^2-2c=0 \Rightarrow c=\dfrac{1+a^2}{2} \Rightarrow \lim\limits_{a\to 0} c=\boxed{\dfrac{1}{2}}$.

---

27. We must have $f'(x)<0$ and $f''(x)<0$. $f'(x)=\int_0^x f''(t)\,dt \Rightarrow f'(x)<0$ for $x>6$.

$f''(x)<0$ for $3<x<9$. The only option that satisfies both conditions is $\boxed{x=8}$.

---

28. If $f'(k\pi)=3\sin(2k\pi)\sin(3k\pi)+2\cos(2k\pi)\cos(3k\pi)=0+2\cdot 1\cdot(-1)=\boxed{-2}$.

---

29. By the Fundamental Theorem of Calculus, $F'(e)=\left.\dfrac{4}{1+\ln x}\right|_{x=e}=\dfrac{4}{1+1}=\boxed{2}$.

---

30. $\dfrac{dy}{dx}=\dfrac{x}{y} \Rightarrow \int y\,dy=\int x\,dx \Rightarrow y^2=x^2+C \Rightarrow y^2-x^2=C$ — $\boxed{\text{a hyperbola}}$.

31.■ $Ae^{0.05\cdot 7}=20000 \Rightarrow A=20000e^{-0.35}=\boxed{14093.76}$.

32.■ $\int_0^y 25e^{0.05t}\,dt=1000 \Rightarrow \left.\dfrac{25}{0.05}e^{0.05t}\right|_0^y=\dfrac{25}{0.05}\left(e^{0.05y}-1\right)=1000 \Rightarrow e^{0.05y}-1=2 \Rightarrow$

$y=20\ln 3\approx 21.97$. The oil reserves will be depleted in $\boxed{2022}$.
