

Be Prepared
for the

AP

Calculus
Exam

Mark Howell

Gonzaga High School, Washington, D.C.

Martha Montgomery

Fremont City Schools, Fremont, Ohio

Practice exam contributors:

Benita Albert

Oak Ridge High School, Oak Ridge, Tennessee

Thomas Dick

Oregon State University

Joe Milliet

St. Mark's School of Texas, Dallas, Texas

Skylight Publishing
Andover, Massachusetts

Copyright © 2011 by Skylight Publishing

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior written permission of the authors and Skylight Publishing.

Library of Congress Control Number: 2011929645

ISBN 978-0-9824775-5-7

Skylight Publishing
9 Bartlet Street, Suite 70
Andover, MA 01810

web: <http://www.skylit.com>
e-mail: sales@skylit.com
support@skylit.com

1 2 3 4 5 6 7 8 9 10 18 17 16 15 14 13 12 11

Printed in the United States of America

Contents

How to Use This Book x

Introduction 1

Chapter 1. Exam Format, Grading, and Tips **3**

- 1.1 Exam Format 3
- 1.2 Grading 5
- 1.3 AB or BC? 8
- 1.4 The Use of Graphing Calculators 9
- 1.5 Exam-Taking Tips 11

Chapter 2. Limits and Continuity **13**

- 2.1 The Concept of Limit 13
- 2.2 Properties of Limits 22
- 2.3 Techniques for Finding Limits 24
 - 2.3.1 Finding Limits Using Continuity 25
 - 2.3.2 Finding Limits of Rational Functions 27
 - 2.3.3 Finding Limits Using Algebraic Techniques 30
 - 2.3.4 Finding Limits Using the Squeeze Theorem 32
 - 2.3.5 Finding Limits that are Derivatives in Disguise 33
 - 2.3.6 L'Hôpital's Rule 35
- 2.4 Continuity 38
- 2.5 Relative Rates of Growth 45
 - Limits and Continuity Worksheet* 49

Chapter 3. Derivatives **53**

- 3.1 Concepts and Notation 53
- 3.2 Differentiation Methods 58
 - 3.2.1 Derivatives of Algebraic Functions 58
 - 3.2.2 Derivatives of Trig, Exponential, and Logarithmic Functions 61
 - 3.2.3 The Chain Rule 63
- 3.3 Higher Order Derivatives 67
- 3.4 Implicit Differentiation 68
- 3.5 The Derivative of the Inverse Function 72
- 3.6 Differentiability and Continuity 74
- 3.7 Finding Derivatives with a Calculator 77
 - Derivatives Worksheet* 79

Chapter 4. Applications of Derivatives **83**


- 4.1 Tangent and Normal Lines and Linear Approximation 83
- 4.2 The Mean Value Theorem 86
- 4.3 Analysis of Function Graphs 90
 - 4.3.1 Increasing and Decreasing Functions 92
 - 4.3.2 Extreme Values (Relative and Absolute) 94
 - 4.3.3 Concavity and Points of Inflection 99
 - 4.3.4 Using the Graphs of f , f' , and f'' in Combination 102
- 4.4 Modeling and Optimization 105
- 4.5 Related Rates Problems 107
- 4.6 Distance, Velocity, Acceleration 112
- 4.7 Data-Driven Problems 115

Chapter 5. Integration **117**

- 5.1 The Fundamental Theorem of Calculus 117
- 5.2 Definite and Indefinite Integrals 119
- 5.3 Approximating Definite Integrals with Sums 121
- 5.4 Finding Limits of Sums Using Definite Integrals 131
- 5.5 Properties of Definite Integrals 133
- 5.6 Calculating Definite Integrals Using Geometry 135
- 5.7 Calculating Definite Integrals Using the FTC 136
- 5.8 Calculating Integrals with a Calculator 139
- 5.9 Finding Antiderivatives 142
 - 5.9.1 General Antidifferentiation Rules 143
 - 5.9.2 Antiderivatives from Known Derivatives 145
 - 5.9.3 u -Substitution (a.k.a. “Change of Variable”) 148
 - 5.9.4 [Antidifferentiation by Parts (BC Only)] 152
 - 5.9.5 [Antidifferentiation by Partial Fractions (BC Only)] 155
- 5.10 [Improper Integrals (BC Only)] 157
 - Integration Worksheet* 165

Chapter 6. Applications of Integrals **171**

- 6.1 Overview 171
- 6.2 Finding the Area of a Region 171
- 6.3 Calculating Volumes 176
 - 6.3.1 Volumes of Solids of Revolution 177
 - 6.3.2 Volumes of Solids with Specified Cross Sections 181
- 6.4 Average Value of a Function 182
- 6.5 Net Change of a Function 184
- 6.6 Motion of a Particle 188
- 6.7 Accumulation Functions 191
- 6.8 [Arc Length (BC Only)] 194

Chapter 7. Differential Equations	197
7.1 What is a Differential Equation?	197
7.2 Slope Fields	198
7.3 Separation of Variables	203
7.4 The Exponential Model	207
7.5 [The Logistic Model (BC Only)]	208
7.6 [Euler's Method (BC Only)]	212
Chapter 8. [Parametric, Vector, and Polar Functions (BC Only)]	219
8.1 Parametric Functions	219
8.2 Vector Functions	227
8.3 Polar Functions	232
Chapter 9. [Series (BC Only)]	239
9.1 The Concept of a Series	239
9.2 Geometric Series	240
9.3 Convergence Tests	241
9.3.1 n -th Term Test	243
9.3.2 Integral Test	244
9.3.3 p -Series	246
9.3.4 Comparison Test	246
9.3.5 Ratio Test	247
9.3.6 Alternating Series Test	247
9.3.7 Applying Convergence Tests	248
9.4 Taylor and Maclaurin Polynomials	252
9.5 Taylor and Maclaurin Series	257
9.6 Taylor's Theorem and Lagrange Remainder	262
Chapter 10. Annotated Solutions to Past Free-Response Questions	267
 http://www.skylit.com/calculus/fr.html	
Appendix: Calculator Skills	269
A.1. Graphing a Function	269
A.2. Solving an Equation	269
A.3. Evaluating a Derivative at a Point	275
A.4. Evaluating an Integral Numerically	277

Practice Exams **281**

- AB-1 283
- AB-2 301
- AB-3 319
- BC-1 339
- BC-2 361

Practice Exams Answers and Solutions **381**

- AB-1 381
- AB-2 389
- AB-3 397
- BC-1 405
- BC-2 415

Index **427**
