

Third Edition

Be Prepared

for the

AP

Calculus Exam

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Introduction

The AP exams in calculus test your understanding of basic concepts and their applications and your fluency with a graphing calculator. There are two different exams at different levels. The AB-level exam covers roughly the material of a one-semester introductory college course in calculus. The BC-level exam adds more advanced material: improper integrals; logistic curves; parametric, vector, and polar functions; and series. BC exam takers receive a subscore that represents their knowledge of AB material. Chapter 1 of this book will help you decide which exam to take.

Exam questions are developed by the College Board's AP Calculus Test Development Committee, and exams are put together by Educational Testing Service, the same organization that administers the SAT and other exams. In 2015, the College Board offered 36 AP exams across all subject areas; 2,483,452 students took 4,478,936 exams (302,532 of them in AP Calculus AB and 118,707 in AP Calculus BC). The most up-to-date information on the available AP exams and participation statistics can be found on the College Board's web site

<http://research.collegeboard.org/programs/ap/data>.

AP Calculus has undergone a major overhaul, with changes to be implemented beginning with the 2017 AP exams. The description of the AB and BC course now follows the Understanding by Design[®] framework developed by Grant Wiggins and Jay McTighe. The *AP Calculus AB and AP Calculus BC Course and Exam Description*, including the *Curriculum Framework*, is available at the College Board's web site.*

For Calculus AB, the framework is structured around three Big Ideas: Limits, Derivatives, and Integrals and the Fundamental Theorem of Calculus (the latter also includes differential equations). [Calculus BC adds a fourth Big Idea, Series.]

This is a lot of material to cover, and it is certainly not the goal of this book to teach you everything you need to know from scratch. For that you need a complete textbook with exercises. Most students who take the exam are enrolled in an AP calculus course at their school. A determined student can prepare for the exam on his or her own; it may take anywhere between three and twelve months, and a good textbook will be even more important.

* (At the time of this book going to print, the 2016-2017 version was posted at <https://advancesinap.collegeboard.org/stem/calculus>.)

The goals of this book are:

- to describe the exam format and requirements;
- to provide an effective review of what you should know, emphasizing the more difficult topics as well as common omissions and mistakes;
- to help you identify and fill the gaps in your knowledge;
- to offer sample exam questions with answers, hints, and solutions to help you practice and analyze your mistakes.

Chapter 1 explains the format, required materials, and rules for using calculators, provides information about exam grading, and offers exam-taking tips. Chapters 2-7 cover the core material: limits and continuity, derivatives and their applications, integrals and their applications, differential equations and slope fields. Chapters 8 and 9 cover more advanced BC-only material: parametric, vector, and polar functions and series. Chapter 10 is actually on the web at this book's companion web site, <http://www.skylit.com/calculus>. It offers annotated solutions to free-response questions from past exams. Chapters 2-9 contain sample multiple-choice and free-response questions with detailed explanations of the right and wrong answers. At the end of the book are five complete practice exams — three AB and two BC, with no overlap — followed by answers and solutions.



The vast majority of the 2017 changes have to do with the ways in which the course content and practices are enumerated, and will not be a major concern for teachers and students. However, there are a few minor content changes, which involve the addition of new topics or explicit mention of some mathematical facts, so that they can be referred to by name on the exam.

The specific content changes that teachers and students need to take note of are the following:

- L'Hospital's Rule is now tested on both the AB and BC exams. Only the indeterminate forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$ will be tested.
- The terms “removable discontinuity”, “jump discontinuity,” and “discontinuity due to a vertical asymptote” have been added to both AB and BC Calculus.
- Antidifferentiation using algebraic techniques, such as long division and completing the square, have been added to both courses. For example, finding $\int \frac{x^2 + x + 1}{x + 1} dx$ can be tested.

┌ And in AP Calculus BC:

- The Limit Comparison Test has been added.
- The Alternating Series Error Bound is now mentioned by name. A question could now be posed like this: “If the first three terms of the Maclaurin series for $\sin x$ were used to approximate $\sin\left(\frac{1}{2}\right)$, what is the alternating series error bound?”
- Absolute and conditional convergence are now specifically mentioned.
- The fact that for an absolutely convergent series one can rearrange the terms in any order without changing the sum is now a part of Calculus BC.

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There are also two changes in the format of the multiple-choice section of the exam. First, each question will have four options instead of five. Second, students will have 60 minutes to complete 30 “closed calculator” questions, followed by 45 minutes to complete 15 “open calculator” questions.

The format of the free-response section of the exams changed in 2011 to two “open calculator” problems followed by four “closed calculator” problems (previously, the split was three open and three closed calculator questions).

This Third Edition of *Be Prepared for the AP Calculus Exam* has been revised to reflect all these changes. We have also added several multiple-choice questions covering the new content to the practice exams. The Calculator Skills appendix has been revised to reflect new models of calculators.

Good luck!