

Be Prepared for the

AP

Calculus Exam

Mark Howell

Gonzaga High School, Washington, D.C.

Martha Montgomery

Fremont City Schools, Fremont, Ohio

Practice exam contributors:

Benita Albert

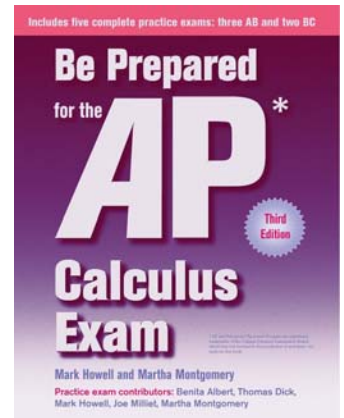
Oak Ridge High School, Oak Ridge, Tennessee

Thomas Dick

Oregon State University

Joe Milliet

St. Mark's School of Texas, Dallas, Texas



Third Edition

* AP and Advanced Placement Program are registered trademarks of the College Entrance Examination Board, which was not involved in the production of and does not endorse this book.

Skylight Publishing
Andover, Massachusetts

Copyright © 2016 by Skylight Publishing

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior written permission of the authors and Skylight Publishing.

Library of Congress Control Number: 2016931566

ISBN 978-0-9972528-5-9

Understanding by Design is a registered trademark of the Association for Supervision and Curriculum Development

Skylight Publishing
9 Bartlet Street, Suite 70
Andover, MA 01810

web: <http://www.skylit.com>
e-mail: sales@skylit.com
support@skylit.com

1 2 3 4 5 6 7 8 9 21 20 19 18 17 16

Printed in the United States of America

Contents

How to Use This Book x

Introduction 1

Chapter 1. Exam Format, Grading, and Tips **5**

- 1.1 Exam Format 5
- 1.2 Grading 7
- 1.3 AB or BC? 10
- 1.4 The Use of Graphing Calculators 11
- 1.5 Exam-Taking Tips 13

Chapter 2. Limits and Continuity **17**

- 2.1 The Concept of Limit 17
- 2.2 Properties of Limits 26
- 2.3 Techniques for Finding Limits 28
 - 2.3.1 Finding Limits Using Continuity 29
 - 2.3.2 Finding Limits of Rational Functions 31
 - 2.3.3 Finding Limits Using Algebraic Techniques 34
 - 2.3.4 Finding Limits Using the Squeeze Theorem 36
 - 2.3.5 Finding Limits That are Derivatives in Disguise 37
 - 2.3.6 L'Hospital's Rule 39
- 2.4 Continuity 42
- 2.5 Relative Rates of Growth 49
 - Limits and Continuity Worksheet* 53

Chapter 3. Derivatives **57**

- 3.1 Concepts and Notation 57
- 3.2 Differentiation Methods 62
 - 3.2.1 Derivatives of Algebraic Functions 62
 - 3.2.2 Derivatives of Trig, Exponential, and Logarithmic Functions 65
 - 3.2.3 The Chain Rule 67
- 3.3 Higher Order Derivatives 71
- 3.4 Implicit Differentiation 72
- 3.5 The Derivative of the Inverse Function 76
- 3.6 Differentiability and Continuity 78
- 3.7 Finding Derivatives with a Calculator 81
 - Derivatives Worksheet* 85

Chapter 4. Applications of Derivatives **89**

- 4.1 Tangent and Normal Lines and Linear Approximation 89
- 4.2 The Mean Value Theorem 92
- 4.3 Analysis of Function Graphs 96
 - 4.3.1 Increasing and Decreasing Functions 98
 - 4.3.2 Extreme Values (Relative and Absolute) 100
 - 4.3.3 Concavity and Points of Inflection 105
 - 4.3.4 Using the Graphs of f , f' , and f'' in Combination 108
- 4.4 Modeling and Optimization 111
- 4.5 Related Rates Problems 113
- 4.6 Distance, Velocity, Acceleration 118
- 4.7 Data-Driven Problems 121

Chapter 5. Integration **123**

- 5.1 The Fundamental Theorem of Calculus 123
- 5.2 Definite and Indefinite Integrals 125
- 5.3 Approximating Definite Integrals with Sums 127
- 5.4 Finding Limits of Sums Using Definite Integrals 137
- 5.5 Properties of Definite Integrals 139
- 5.6 Calculating Definite Integrals Using Geometry 141
- 5.7 Calculating Definite Integrals Using the FTC 142
- 5.8 Calculating Integrals with a Calculator 145
- 5.9 Finding Antiderivatives 148
 - 5.9.1 General Antidifferentiation Rules 149
 - 5.9.2 Antiderivatives from Known Derivatives 151
 - 5.9.3 u -Substitution (a.k.a. “Change of Variable”) 154
 - 5.9.4 Antidifferentiation by Long Division and Completing the Square 158
 - 5.9.5 [Antidifferentiation by Parts (BC Only)] 161
 - 5.9.6 [Antidifferentiation by Partial Fractions (BC Only)] 164
- 5.10 [Improper Integrals (BC Only)] 166
Integration Worksheet 173

Chapter 6. Applications of Integrals **179**

- 6.1 Overview 179
- 6.2 Finding the Area of a Region 179
- 6.3 Calculating Volumes 184
 - 6.3.1 Volumes of Solids of Revolution 185
 - 6.3.2 Volumes of Solids with Specified Cross Sections 189
- 6.4 Average Value of a Function 190
- 6.5 Net Change of a Function 192
- 6.6 Motion of a Particle 196
- 6.7 Accumulation Functions 199
- 6.8 [Arc Length (BC Only)] 202

Chapter 7. Differential Equations	205
7.1 What is a Differential Equation?	205
7.2 Slope Fields	206
7.3 Separation of Variables	211
7.4 The Exponential Model	215
7.5 「 The Logistic Model (BC Only) 」	216
7.6 「 Euler's Method (BC Only) 」	220
Chapter 8. 「 Parametric, Vector, and Polar Functions (BC Only) 」	227
8.1 Parametric Functions	227
8.2 Vector Functions	235
8.3 Polar Functions	240
Chapter 9. 「 Series (BC Only) 」	247
9.1 The Concept of a Series	247
9.2 Arithmetic Properties of Series	248
9.3 A Necessary Condition for Convergence	249
9.4 Absolute and Conditional Convergence	250
9.5 Geometric Series	251
9.6 Convergence Tests	252
9.6.1 n -th Term Test	253
9.6.2 Ratio Test	253
9.6.3 Integral Test	254
9.6.4 p -Series	256
9.6.5 Comparison Test	256
9.6.6 Limit Comparison Test	257
9.6.7 Applying Convergence Tests	258
9.7 Alternating Series	261
9.8 Power Series	262
9.9 Taylor and Maclaurin Polynomials	264
9.10 Taylor and Maclaurin Series	268
9.11 Taylor's Theorem and Lagrange Remainder	273
9.12 Error Bound for Alternating Taylor Series	276
Chapter 10. Annotated Solutions to Past Free-Response Questions	279
✧ http://www.skylit.com/calculus/fr.html	
Appendix: Calculator Skills	279
✧ http://www.skylit.com/calculus/	

Practice Exams **281**

- AB-1 281
- AB-2 299
- AB-3 319
- BC-1 341
- BC-2 361

Practice Exams Answers and Solutions **383**

- AB-1 383
- AB-2 391
- AB-3 399
- BC-1 407
- BC-2 415

Index **427**
